# V. *On the Air-Engine.* *By* James Prescott Joule, *F.R.S., F.C.S., Corr. Mem. R.A. Turin, Sec. Lit. and Phil. Soc. Manchester, &c.*

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It has long been suspected that important advantages might be derived from the substitution of air for steam as a prime mover of machinery. It has been alleged that the air-engine would be safer, lighter, and more economical in the expenditure of fuel than the steam-engine. Until comparatively recent times, however, experimental science was hardly in the state of advancement requisite to enable the physicist, in his investigation of this important subject, to arrive at conclusions sufficiently certain to give confidence to the practical machinist. Professor Thomson, Mr. Rankine, and M. Clausius have of late, however, published papers of great value on the mechanical action of gases, and particularly of steam, founded on tolerably correct experimental data. I hope that the following remarks founded on the same general principles, but applied to a particular kind of air-engine, may be interesting to the Royal Society.

The air-engine, the performance of which I propose to discuss, consists of two parts, in one of which the air is compressed into a receiver, where its elasticity is increased by the application of heat, and in the other it is allowed to escape again from the receiver into the atmosphere. By the former work is absorbed, by the latter it is evolved in a larger quantity, the excess constituting the work evolved by the engine on the whole. The simple question, therefore, is to determine the quantity of work so evolved, together with the heat applied to increase the elasticity of the air in the receiver.

In Plate VI. fig. 1 let A be the pump by which air is forced into the receiver C, where heat may be communicated to it from an external source, and B the cylinder, by which the same quantity is allowed to escape again into the atmosphere. Moreover, let the material of which the apparatus is made, with the exception of that part through which heat may be communicated to the air in C, be impervious to, and destitute of capacity for heat. Such a machine may be conceived to work in the following manner.

The cylinder of the pump A being filled with air of the atmospheric temperature and pressure, the piston compresses the air until, at a point *n*, its pressure is rendered equal to that of the air in the receiver C, which has been previously filled with air of an elevated temperature and pressure. The work absorbed by this action will be that communicated to the air in the cylinder, minus the work due to the atmospheric



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pressure through $mn$. The moment the piston has passed the point $n$ the valve will open, admitting the air into the receiver C; and as this receiver may be conceived to be of indefinite magnitude, the alteration of pressure in it, consequent upon the introduction of fresh air, may be neglected. Heat is then communicated to the air in the receiver, in order to restore its temperature to the intensity which existed before the admission of air at a lower temperature. The air is then allowed to escape from the receiver into the base of the cylinder B, evolving work until, on the arrival of the piston at $n'$, the same quantity has been removed from the receiver as was forced into it by the pump. The further supply of air from the receiver is then cut off, and that which has entered the cylinder expands, evolving work until, on the arrival of the piston at $m'$, its pressure is reduced to that of the atmosphere. By opening valves at the bases of A and B, the pistons are then brought to their first positions.

The problem which must be solved in order to estimate the power and consumption of fuel in an engine similar to that just described, is as follows:—To determine the pressure and temperature for any point of the stroke of a piston which compresses a given volume of air, and the quantity of work absorbed in forcing the piston to that point. For the temperature and pressure POISSON has furnished the following formulæ,—

$$\frac{T'}{T}=\left(\frac{V}{V'}\right)^{k-1},$$

and

$$\frac{P'}{P}=\left(\frac{V}{V'}\right)^{k},$$

where T, P, and V are the temperature from absolute zero (estimated at 491° FAHR. below the freezing-point of water), pressure, and volume of the air before compression; T′, P′, and V′ the temperature from absolute zero, pressure, and volume of air after compression; and $k$ is the ratio of the specific heat of air at constant pressure to that at constant volume. Professor W. THOMSON has deduced, as a consequence of the above, the following formula for the work absorbed,

$$W=PV\frac{1}{k-1}\left\{\left(\frac{V}{V'}\right)^{k-1}-1\right\}*.$$

From the foregoing formulæ I have calculated the work absorbed by compressing air in a cylinder 1 foot long, and of the capacity of 12 cubic inches, the absolute temperature of the air, and its pressure at each tenth of an inch of the piston's progress. The following data were employed in the computation:—Weight of 100 cubic inches of atmospheric air of 15 lbs. pressure on the square inch, and 491°

* The above formula was kindly communicated to the author by Professor THOMSON, in a letter dated January 15, 1851, from which the following is an extract:—"It is required to find the work necessary to compress a given mass of air to a given fraction of its volume, when no heat is permitted to leave the air. Let P, V, T be the primitive pressure, volume, and temperature, respectively; let $p$, $v$, and $t$ be the pressure, volume, and temperature at any instant during the compression; and let P′, V′, and T′ be what they become

FAHR. from the absolute zero, 33·2237 grs.; specific heat of air at constant volume, 0·19742. Ratio of the specific heat of air at constant pressure to that at constant volume, as determined from the experiments of DELAROCHE and BERARD, and the mechanical equivalent of heat, 1·3519325*. The results are shown in Table I.

I now proceed to give some estimates of the performance of an air-engine similar in principle to that already described, worked at various pressures and temperatures, those of the atmospheric air being 15 lbs. on the square inch, and 32° FAHR. or 491° FAHR. from the absolute zero. In order to render the results easily available in calculating the duty of engines of greater size, I shall assume that the condensing pump is 12 inches long, and has a sectional area equal to 1 square inch, and that the cylinder, also of 1 inch section, has a length which may be made to vary according to the pressure and temperature employed.

I take as the first example, a case in which the receiver C contains air of the atmospheric density, and of which the absolute temperature is 849°·464 FAHR. or 390°·464 of the scale of FAHRENHEIT's thermometer. The pressure in the receiver will then be 25·95104 lbs. on the square inch, as given in the third column of Table II. The air in the pump A will be brought to the same pressure, and to the absolute temperature 566°·3094 after the piston has traversed 4 inches. The work absorbed by the air will be 6·537154 foot-pounds, from which, by subtracting 5 foot-pounds, the work communicated by the pressure of the atmosphere following the piston, we obtain 1·537154 foot-pounds as the work of the engine absorbed by the first part of the stroke. This result is consigned to column 6. Immediately after the piston has passed the fourth inch of the pump, the valve will be opened admitting the compressed air into the receiver C. The work of the engine absorbed by the re-

when the compression is concluded. Then if $k$ denote the ratio of the specific heat of air at constant pressure to the specific heat of air kept in a space of constant volume, and if, as appears to be nearly, if not rigorously true, $k$ be constant for varying temperatures and pressures, we shall have by the investigation in MILLER's 'Hydrostatics' (Edit. 1835, p. 22)—

$$\frac{1+Et}{1+ET}=\left(\frac{V}{v}\right)^{k-1}.$$

But

$$\frac{pv}{PV}=\frac{1+Et}{1+ET},$$

therefore

$$pv=PV\left(\frac{V}{v}\right)^{k-1}.$$

Now the work done in compressing the mass from volume $v$ to volume $v-dv$ will be $pdv$, or by what precedes,

$$PV\,.\,V^{k-1}\frac{dv}{v^k}.$$

Hence by the integral calculus we readily find, for the work, W, necessary to compress from V to V′,

$$W=PV\,.\,\frac{1}{k-1}\left\{\left(\frac{V}{V'}\right)^{k-1}-1\right\}."$$

* The experiments of DESORMES and CLEMENT give 1·354; those of GAY-LUSSAC and WELTER 1·375; and those described under the article 'Hygrometry' (Enc. Brit.), 1·333. See Art. 'Sound,' Enc. Brit., 7th Edit.

maining 8 inches of the piston's stroke will be $\frac{8}{12}$ (25·95104−15)=7·300693 foot-pounds, as given in the seventh column. The air thus forced into the receiver at the absolute temperature 566°·3094 FAHR. must then be raised to 849°·464 FAHR., the constant absolute temperature of the receiver. The heat necessary for this purpose, being that due to the capacity for heat of air at constant pressure, will be that which is able to raise the temperature of 1 lb. of water 0°·04304312 FAHR., as given in column 15. On leaving the receiver, the air enters the cylinder of expansion B, and having propelled the piston through 12 inches, the same quantity of air will have passed out of the receiver as was pumped into it by A. The further supply of air is then cut off, and the air after expanding through the remaining 6 inches of the cylinder (which in this case must be 18 inches long), will be reduced to the pressure of 15 lbs. on the square inch, and the absolute temperature $\frac{3}{2}$ (491°)=736°·5. The work evolved by the piston will also be to that absorbed in the condensing pump, as the volume of the cylinder B is to that of the pump A; from which we find $\frac{3}{2}$ (7·300693)=10·95104 foot-pounds, and $\frac{3}{2}$ (1·537154)=2·305731 foot-pounds, the work evolved by the first and second parts of the piston's stroke, as given in columns 11 and 12. The work evolved by the engine on the whole, being the difference between the work evolved by B, and the work absorbed by A, will be equal to one-third of the former, or one-half of the latter, or 4·418924 foot-pounds, as given in column 14. Dividing this by 0°·04304312, we obtain 102·66276 foot-pounds as the work evolved by the engine out of each 1° FAHR. per lb. of water communicated to the receiver. This result, which is consigned to the sixteenth column, informs us of the economical value of the engine, which is of course great in proportion to its approach to 772 foot-pounds, the theoretical maximum. The seventeenth column contains the theoretical duty according to Professor THOMSON's law, viz. that the range of temperature divided by the maximum absolute temperature is equal to the fraction of heat converted into force by any perfect engine*.

It will be observed that the numbers in column 16, representing the work evolved out of each unit of heat, increase with the temperature and pressure of the air in the receiver. In every example given, with the exception of the first, the economical value of the air-engine in question is greater than that of the steam-engine calculated by Mr. RANKINE in his paper on the Mechanical Action of Heat†. In considering the relative merits of the engines, we must not, however, lose sight of a most important fact discovered by RANKINE and CLAUSIUS, viz. that a portion of the heat

* See Professor THOMSON's "Investigation of the Duty of a perfect Thermo-Dynamic Engine," at the end of this paper.

† Transactions of the Royal Society of Edinburgh, vol. xx. part 1. Professor THOMSON, in a paper "On the Dynamical Theory of Heat," recently read before the Royal Society, Edinburgh, gives 209 foot-pounds as the duty of an absolutely perfect steam-engine, with a range of temperature between 30° and 140° Centigrade.

## Table I.

| Distance traversed by piston, in inches. | Work absorbed, in foot-pounds. | Temperature from absolute zero, in degrees Fahr. | Pressure on the piston, in lbs. | Distance traversed by piston, in inches. | Work absorbed, in foot-pounds. | Temperature from absolute zero, in degrees Fahr. | Pressure on the piston, in lbs. |
|---|---|---|---|---|---|---|---|
| 0 | 0· | 491 | 15 | 6·0 | 11·77479 | 626·6480 | 38·28805 |
| 0·1 | 0·1257008 | 492·4481 | 15·17066 | 6·1 | 12·09828 | 630·3747 | 39·16857 |
| 0·2 | 0·2528426 | 493·9128 | 15·34473 | 6·2 | 12·42768 | 634·1694 | 40·08375 |
| 0·3 | 0·3814514 | 495·3944 | 15·52230 | 6·3 | 12·76566 | 638·0630 | 41·03738 |
| 0·4 | 0·5113882 | 496·8913 | 15·70343 | 6·4 | 13·11172 | 642·0498 | 42·03120 |
| 0·5 | 0·6432696 | 498·4106 | 15·88841 | 6·5 | 13·46626 | 646·1341 | 43·06763 |
| 0·6 | 0·7763749 | 499·9440 | 16·07709 | 6·6 | 13·82962 | 650·3201 | 44·14936 |
| 0·7 | 0·9111464 | 501·4966 | 16·26974 | 6·7 | 14·20221 | 654·6124 | 45·27926 |
| 0·8 | 1·047533 | 503·0678 | 16·46643 | 6·8 | 14·58441 | 659·0155 | 46·46043 |
| 0·9 | 1·185586 | 504·6582 | 16·66731 | 6·9 | 14·97668 | 663·5345 | 47·69625 |
| 1·0 | 1·325341 | 506·2682 | 16·87248 | 7·0 | 15·37948 | 668·1749 | 48·99042 |
| 1·1 | 1·466805 | 507·8979 | 17·08209 | 7·1 | 15·79334 | 672·9426 | 50·34692 |
| 1·2 | 1·610032 | 509·5479 | 17·29627 | 7·2 | 16·21879 | 677·8438 | 51·77015 |
| 1·3 | 1·755090 | 511·2190 | 17·51516 | 7·3 | 16·65633 | 682·8845 | 53·26481 |
| 1·4 | 1·901962 | 512·9110 | 17·73892 | 7·4 | 17·10672 | 688·0730 | 54·83624 |
| 1·5 | 2·050727 | 514·6248 | 17·96770 | 7·5 | 17·57047 | 693·4156 | 56·49008 |
| 1·6 | 2·201444 | 516·3611 | 18·20167 | 7·6 | 18·04842 | 698·9216 | 58·23268 |
| 1·7 | 2·354089 | 518·1196 | 18·44097 | 7·7 | 18·54128 | 704·5995 | 60·07098 |
| 1·8 | 2·508791 | 519·9018 | 18·68582 | 7·8 | 19·04988 | 710·4585 | 62·01267 |
| 1·9 | 2·665542 | 521·7076 | 18·93638 | 7·9 | 19·57510 | 716·5093 | 64·06620 |
| 2·0 | 2·824402 | 523·5377 | 19·19283 | 8·0 | 20·11797 | 722·7632 | 66·24102 |
| 2·1 | 2·985424 | 525·3927 | 19·45538 | 8·1 | 20·67947 | 729·2318 | 68·54755 |
| 2·2 | 3·148667 | 527·2733 | 19·72426 | 8·2 | 21·26078 | 735·9287 | 70·99750 |
| 2·3 | 3·314184 | 529·1801 | 19·99966 | 8·3 | 21·86317 | 742·8683 | 73·60395 |
| 2·4 | 3·481993 | 531·1133 | 20·28182 | 8·4 | 22·48795 | 750·0659 | 76·38145 |
| 2·5 | 3·652224 | 533·0744 | 20·57099 | 8·5 | 23·13667 | 757·5393 | 79·34657 |
| 2·6 | 3·824870 | 535·0633 | 20·86740 | 8·6 | 23·81096 | 765·3072 | 82·51786 |
| 2·7 | 4·000022 | 537·0811 | 21·17132 | 8·7 | 24·51263 | 773·3907 | 85·91642 |
| 2·8 | 4·177719 | 539·1282 | 21·48302 | 8·8 | 25·24353 | 781·8109 | 89·56590 |
| 2·9 | 4·358080 | 541·2060 | 21·80279 | 8·9 | 26·00607 | 790·5954 | 93·49398 |
| 3·0 | 4·541123 | 543·3147 | 22·13095 | 9·0 | 26·80260 | 799·7716 | 97·73172 |
| 3·1 | 4·726945 | 545·4554 | 22·46778 | 9·1 | 27·63584 | 809·3707 | 102·3153 |
| 3·2 | 4·915606 | 547·6288 | 22·81364 | 9·2 | 28·50889 | 819·4284 | 107·2862 |
| 3·3 | 5·107206 | 549·8361 | 23·16887 | 9·3 | 29·42512 | 829·9836 | 112·6929 |
| 3·4 | 5·301779 | 552·0776 | 23·53383 | 9·4 | 30·38842 | 841·0810 | 118·5920 |
| 3·5 | 5·499456 | 554·3549 | 23·90892 | 9·5 | 31·40316 | 852·7710 | 125·0499 |
| 3·6 | 5·700287 | 556·6685 | 24·29452 | 9·6 | 32·47432 | 865·1110 | 132·1452 |
| 3·7 | 5·904370 | 559·0196 | 24·69107 | 9·7 | 33·60755 | 878·1660 | 139·9716 |
| 3·8 | 6·111816 | 561·4094 | 25·09902 | 9·8 | 34·80946 | 892·0122 | 148·6412 |
| 3·9 | 6·322706 | 563·8389 | 25·51884 | 9·9 | 36·08755 | 906·7362 | 158·2897 |
| 4·0 | 6·537154 | 566·3094 | 25·95104 | 10·0 | 37·45073 | 922·4402 | 169·0827 |
| 4·1 | 6·755242 | 568·8218 | 26·39613 | 10·1 | 38·90927 | 939·2430 | 181·2239 |
| 4·2 | 6·977122 | 571·3779 | 26·85467 | 10·2 | 40·47547 | 957·2860 | 194·9666 |
| 4·3 | 7·202863 | 573·9785 | 27·32725 | 10·3 | 42·16396 | 976·7377 | 210·6300 |
| 4·4 | 7·432590 | 576·6250 | 27·81448 | 10·4 | 43·99234 | 997·8010 | 228·6204 |
| 4·5 | 7·666465 | 579·3193 | 28·31703 | 10·5 | 45·98215 | 1020·724 | 249·4640 |
| 4·6 | 7·904577 | 582·0624 | 28·83559 | 10·6 | 48·15980 | 1045·811 | 273·8522 |
| 4·7 | 8·147090 | 584·8562 | 29·37090 | 10·7 | 50·55854 | 1073·445 | 302·7106 |
| 4·8 | 8·394129 | 587·7021 | 29·92373 | 10·8 | 53·22073 | 1104·114 | 337·3058 |
| 4·9 | 8·645876 | 590·6023 | 30·49494 | 10·9 | 56·20106 | 1138·448 | 379·4122 |
| 5·0 | 8·902414 | 593·5577 | 31·08536 | 11·0 | 59·57200 | 1177·282 | 431·5900 |
| 5·1 | 9·164006 | 596·5713 | 31·69598 | 11·1 | 63·43234 | 1221·754 | 497·6597 |
| 5·2 | 9·430732 | 599·6440 | 32·32776 | 11·2 | 67·92089 | 1273·463 | 583·5623 |
| 5·3 | 9·702832 | 602·7787 | 32·98178 | 11·3 | 73·23965 | 1334·736 | 699·0180 |
| 5·4 | 9·980470 | 605·9771 | 33·65917 | 11·4 | 79·69880 | 1409·147 | 860·9854 |
| 5·5 | 10·26387 | 609·2420 | 34·36112 | 11·5 | 87·80466 | 1502·528 | 1101·650 |
| 5·6 | 10·55323 | 612·5754 | 35·08897 | 11·6 | 98·46010 | 1625·281 | 1489·565 |
| 5·7 | 10·84876 | 615·9800 | 35·84405 | 11·7 | 113·4919 | 1798·450 | 2197·699 |
| 5·8 | 11·15067 | 619·4581 | 36·62784 | 11·8 | 137·4363 | 2074·295 | 3802·170 |
| 5·9 | 11·45928 | 623·0133 | 37·44196 | 11·9 | 187·1806 | 2647·359 | 9705·187 |

# Table II.

| No. of Example. | Receiver C. | | | Pump of Compression A. Length 12 inches. Sectional Area = 1 square inch. | | | | Cylinder of Expansion B. Sectional Area = 1 square inch. | | | | | Work evolved by the engine by each stroke of the piston, in foot-pounds. | Heat communicated to the air in receiver C, in degrees Fahr. per capacity of a lb. of water. | Work evolved out of each degree Fahr. in the capacity of a lb. of water, in foot-pounds. | Difference between the numbers in columns 4 and 13, divided by the numbers in column 4, and multiplied by the mechanical equivalent of heat. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Density of the air, that of the atmosphere being called unity. | Pressure of the air in lbs. on the square inch. | Absolute temperature of the air, in degrees Fahr. from the absolute zero. | Length of the first part of the stroke. | Work of the engine absorbed by the first part of the stroke of the piston, in foot-pounds. | Work of the engine absorbed by the second part of the stroke of the piston, in foot-pounds. | Absolute temperature of the air forced into receiver C, in degrees Fahr. from the absolute zero. | Length of cylinder B, in inches. | Length of the first part of the piston's stroke, in inches. | Work communicated to the engine by the first part of the stroke of the piston, in foot-pounds. | Work communicated to the engine by the second part of the stroke, in foot-pounds. | Absolute temperature of the air escaping into the atmosphere, in degrees Fahr. from absolute zero. | | | | |
| 1 | 1 | 25·95104 | 849·464 | 4 | 1·537154 | 7·300693 | 566·3094 | 18 | 12 | 10·95104 | 2·305731 | 736·5 | 4·418924 | 0·04304312 | 102·6628 | 102·6626 |
| 2 | 1 | 66·24102 | 2168·289 | 8 | 10·11797 | 17·08034 | 722·7632 | 36 | 12 | 51·24102 | 30·35391 | 1473·0 | 54·39662 | 0·2197384 | 247·5517 | 247·5515 |
| 3 | 2 | 66·24102 | 1084·145 | 8 | 10·11797 | 17·08034 | 722·7632 | 18 | 6 | 25·62051 | 15·17695 | 736·5 | 13·59915 | 0·0549346 | 247·5517 | 247·5515 |
| 4 | 2 | 169·0827 | 2767·321 | 10 | 24·95073 | 25·68045 | 922·4402 | 36 | 6 | 77·04135 | 74·85219 | 1473·0 | 101·2624 | 0·2804454 | 361·0769 | 361·0770 |
| 5 | 4 | 169·0827 | 1383·660 | 10 | 24·95073 | 25·68045 | 922·4402 | 18 | 3 | 38·52067 | 37·42610 | 736·5 | 25·31569 | 0·07011137 | 361·0769 | 361·0770 |
| 6 | 8 | 431·59 | 1765·923 | 11 | 45·82200 | 34·71583 | 1177·282 | 18 | 1·5 | 52·07375 | 68·73300 | 736·5 | 40·26892 | 0·08948094 | 450·0278 | 450·0279 |
| 7 | 20 | 1101·65 | 1803·034 | 11·5 | 73·42966 | 45·27708 | 1502·528 | 14·4 | 0·6 | 54·33250 | 88·11559 | 589·2 | 23·74135 | 0·04568073 | 519·7236 | 519·7238 |
| 8 | 100 | 9705·187 | 3176·831 | 11·9 | 172·3056 | 80·75156 | 2647·359 | 14·4 | 0·12 | 96·90187 | 206·7667 | 589·2 | 50·61143 | 0·0804865 | 628·8189 | 628·8189 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |

employed to evaporate water in the boiler is afterwards evolved in the form of work, in consequence of the liquefaction, in the cylinder, of a portion of the expanding vapour. This fact would induce the hope that a great portion of the latent heat of evaporation, which is at present almost entirely lost, might by an increase of temperature, and by extending the principle of expansion, be converted into mechanical effect.

If, as would appear from the experiments of De la Rive and Marcet, Haycraft and Dulong, the capacity for heat of a given volume is the same in all gases taken at the same pressure and temperature, the results of the above Tables will be equally true whatever elastic fluid be employed.

It now only remains to offer a few observations, with a view to facilitate the labours of those who may be desirous of constructing a good practical air-engine.

It may be remarked, in the first place, that the receiver C need not be of much greater capacity than the cylinder B. For in the reciprocating engine, the air could be introduced from the pump A, at the same time that an equal amount would be expelled into the cylinder B. It would therefore be only requisite to pass the air through tubes heated by a proper furnace, as in Neilson's *hot-blast*, the tubes themselves constituting the receiver C. For a temperature under the red heat, these tubes might be constructed of wrought or cast iron. They might be either straight, like the tubes of a locomotive boiler, or arranged in the form of a coil, as represented by fig. 2, in which *a* is the pipe which conveys the air from the pump, *c*, *c*, *c*, &c. is the coil of wrought or cast-iron tubing, and *b* is the pipe which conveys the heated air to the cylinder. The coil is surrounded by a massive arch of brickwork, which serves at once to support the pipes, and to prevent waste of heat. To prevent the temperature exceeding the proper limits, the pipe *b* might, as it expands by the heat of the inclosed air, move a piece of mechanism in connection with the damper of the flue. I may remark that, on the scales adopted, fig. 2 represents the size of receiver which would be required for an engine the cylinder of which is 3 feet in diameter.

I would here venture to suggest whether the combustion of the fuel could not, by suitable mechanical arrangements, be carried on within the receiver C; if this could be accomplished, the heat, which in the form of receiver already described is lost up the chimney, would be economized, and a great saving of weight and space would be effected. An engine furnished with a receiver of this kind would be strikingly analogous to the electro-magnetic engine, and present a beautiful illustration of the evolution of mechanical effect from chemical forces.

In both of the above forms of receiver, it would be desirable, as already hinted, that the introduction of the air into the receiver should be simultaneous with the expulsion of the same quantity into the cylinder. This is necessary in order both to keep the pressure in the receiver uniform and to promote the smooth action of the engine. For this purpose the piston-rods of the pump and cylinder, *a* and *b* (fig. 3), must be attached to cranks on different parts of the circumference of the revolving shaft *c c*, so contrived that the piston shall arrive at the top or bottom of the cylinder the moment that the pump-valve opens admitting a fresh supply of air into the receiver. The cylinder should of course be provided with proper expansion gear to cut off the air at the required part of the stroke, which must be a constant quantity for each engine. The valves of the pump would of course be self-acting.

In an engine similar to that described, it will be obvious that if the temperature of the receiver be kept constant, the pressure of air in it will also remain constant. For whilst the same quantity of air is always introduced into the receiver by each stroke of the pump, the quantity expelled out of it would increase with an augmentation and decrease with a diminution of pressure.

In conclusion, I would recommend the examples No. 3 and No. 5 of Table II. to the attention of those who may be willing to construct an air-engine. In both of these cases the capacity of the pump is two-thirds of that of the cylinder. In the cylinder of No. 3 the air is to be cut off at one-third of the stroke; and in that of No. 5 at one-sixth of the stroke. The temperature of the air in the receiver (supposing that of the atmosphere to be 32° FAHR.) is 625°·145 FAHR. in No. 3, and 924°·66 FAHR. in No. 5. The consumption of fuel in No. 3 need not exceed one-half, nor that in No. 5 one-third of that in the most perfect steam-engines at present constructed.

*Acton Square, Salford, Manchester,*
*May* 6, 1851.

## *Note to the foregoing Paper, with a New Experimental Determination of the Specific Heat of Atmospheric Air.*

Received March 23, 1852.

Since the above was written, Professor W. H. MILLER has directed my attention to the probable incorrectness of the value of $k$, as deduced from the experiments of DELAROCHE and BERARD on the specific heat of air, and my own determination of the mechanical equivalent of heat; in comparison with the value deduced from the numerous and excellent experiments on the velocity of sound. Mr. RANKINE considers that the discrepancy between the two values arises from the incorrectness of DELAROCHE and BERARD's result, an opinion which would seem to be justified by the entire want of accordance between the determination of these philosophers, and those of SUERMANN, and CLEMENT and DESORMES. I have therefore been induced to make the following careful experiments in order to obtain a fresh and, if possible, more correct value of the specific heat of air at constant pressure.

The apparatus I employed is represented by fig. 4, in which $a$ and $b$ are two vessels, each of which contains a coil of leaden piping, eight yards long and one quarter of an inch in internal diameter. The coil of the upper vessel passes three-eighths of an inch through the bottom, to which it is soldered at $c$, and is thence connected with the coil of the lower vessel by a piece of vulcanized india rubber tubing. This part of the apparatus will be better understood by a reference to fig. 5, in which a section of it is represented, *a* being the upper, *b* the lower vessel, and *w* the surface of the water in the latter. *x x* are a pair of wooden pincers by means of which the india rubber tube could be compressed so as to prevent, when desired, any communication between the air in the two coils of piping. Referring again to fig. 4, *g* is a gas-lamp to maintain the water in the upper vessel at a constant high temperature, and *j* is a tall jar filled with coarsely pounded chloride of calcium, in passing through which the air was entirely deprived of aqueous vapour; a length of vulcanized india rubber tubing, *p*, connects the coil of the lower vessel with a good air-pump, each barrel of which was found to have a capacity of 12·77 cubic inches. The temperature of the pump could be ascertained by means of a small thermometer, the bulb of which was kept in contact with one of the barrels.

The method of experimenting was as follows:—The lower vessel being filled with cold water, and the upper with water raised to about 190°, their exact temperatures were read off, with the usual precautions, from the scales of delicate and accurate thermometers. The pump was then worked at a uniform velocity for twenty-six minutes, the water in the lower vessel being agitated from time to time by a stirrer. The examination of the barometer and thermometers a second time occupied four minutes more; so that the whole time occupied by each experiment was exactly half an hour. The pincers were now applied so as to cut off all communication between the air in the two coils, and the effect of the various causes of a change of temperature in the lower vessel, unconnected with the current of heated air, was observed during another half-hour. Experiments of both the above kinds were repeated several times with the results tabulated below.

I may remark in this place that I had ascertained, by preliminary experiments, that the air passed from the coils of the vessels sensibly at the temperatures registered by the thermometers plunged into the surrounding water.

Series I.—Pump worked 26′, at the rate of twenty-four strokes per minute.

| No. of Experiment. | Source of calorific effect. | Height of Barometer. | Temperature of Barometer. | Temperature of Air-pump. | Temperature of upper vessel. | Temperature of the room. | Temperature of the lower vessel. Commencement of Experiment. | Temperature of the lower vessel. Termination of Experiment. | Increase of temperature. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Radiation | ........ | ° | ° | ° | 46·081 (°) | 41·270 (°) | 41·814 (°) | 0·544 (°) |
| 1 | Heated air and radiation | 30·195 | 46 | 49·3 | 189·28 | 46·188 | 41·814 | 42·802 | 0·988 |
| 2 | Radiation | ........ | ........ | ........ | ........ | 46·497 | 42·802 | 43·304 | 0·502 |
| 2 | Heated air and radiation | 30·205 | 46·75 | 50·3 | 189·43 | 46·785 | 43·304 | 44·246 | 0·942 |
| 3 | Radiation | ........ | ........ | ........ | ........ | 46·948 | 44·246 | 44·694 | 0·448 |
| 3 | Heated air and radiation | 30·22 | 47·5 | 51·1 | 189·89 | 47·068 | 44·694 | 45·590 | 0·896 |
| 4 | Radiation | ........ | ........ | ........ | ........ | 47·197 | 45·590 | 45·983 | 0·393 |
| 4 | Heated air and radiation | 30·235 | 48· | 51·7 | 194·85 | 47·283 | 45·983 | 46·856 | 0·873 |
| 5 | Radiation | ........ | ........ | ........ | ........ | 47·455 | 46·856 | 47·211 | 0·355 |
| Mean. | Heated air and radiation | 30·214 | 47·06 | 50·6 | 190·862 | 46·831 | 43·949 | 44·874 | 0·925 |
| Mean. | Radiation | ........ | ........ | ........ | ........ | 46·836 | 44·153 | 44·601 | 0·448 |

It will be observed that the excess of the temperature of the room above the mean temperature of the water in the lower vessel, was, in the experiments with heated air, $2^{\circ}\cdot42$, but in the experiments on the effect of radiation $2^{\circ}\cdot459$. A comparison of the several experiments with one another, furnished the means of determining the amount of the small correction due to this circumstance. Hence $0^{\circ}\cdot925+0^{\circ}\cdot002-0^{\circ}\cdot448=0^{\circ}\cdot479$ will be the corrected mean increase of temperature due to the current of heated air. The material in which this increase took place consisted of 175500 grs. of water, 15635 grs. of copper, and 53370 grs. of lead, the whole having a capacity for heat equivalent to that of 178535 grs. of water. The volume of air passed through the pump was $12\cdot77\times26\times24=7968\cdot48$ cubic inches, which, at the observed barometric pressure and the temperature $50^{\circ}\cdot6$, would weigh 2537·94 grs. We have therefore for the specific heat of atmospheric air at constant pressure—

$$\frac{178535\times0\cdot479}{2537\cdot94\times146\cdot45}=0\cdot23008.$$

SERIES II.—Pump worked 26′, at the rate of forty strokes per minute.

| No. of Experiment. | Source of calorific effect. | Height of Barometer. | Temperature of Barometer. | Temperature of Air-pump. | Temperature of upper vessel. | Temperature of the room. | Temperature of the lower vessel. | | Increase of temperature. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Commencement of Experiment. | Termination of Experiment. | |
| 1 | Radiation........................ | ........ | °........ | °........ | °............ | 47·223 | 44·200 | 44·648 | 0·448 |
| 1 | Heated air and radiation ... | 30·6 | 47·75 | 52 | 197·71 | 47·558 | 44·648 | 45·902 | 1·254 |
| 2 | Radiation........................ | ........ | ........ | ........ | ............ | 47·841 | 45·902 | 46·319 | 0·417 |
| 2 | Heated air and radiation ... | 30·602 | 48·25 | 53·5 | 198·63 | 48·099 | 46·319 | 47·516 | 1·197 |
| 3 | Radiation........................ | ........ | ........ | ........ | ............ | 48·339 | 47·516 | 47·860 | 0·344 |
| 3 | Heated air and radiation ... | 30·61 | 49·5 | 55·4 | 202·42 | 49·107 | 49·327 | 50·443 | 1·116 |
| 4 | Radiation........................ | ........ | ........ | ........ | ............ | 49·524 | 50·443 | 50·728 | 0·285 |
| 4 | Heated air and radiation ... | 30·607 | 50·25 | 56·4 | 203·13 | 49·850 | 50·728 | 51·809 | 1·081 |
| 5 | Radiation........................ | ........ | ........ | ........ | ............ | 50·030 | 51·809 | 52·037 | 0·228 |
| Mean. | Heated air and radiation ... | 30·605 | 48·94 | 54·32 | 200·472 | 48·653 | 47·755 | 48·917 | 1·162 |
| Mean. | Radiation........................ | ........ | ........ | ........ | ............ | 48·591 | 47·974 | 48·318 | 0·344 |

In the above series $1^{\circ}\cdot162+0^{\circ}\cdot006-0^{\circ}\cdot344=0^{\circ}\cdot824$ will be the corrected mean increase of temperature due to the current of heated air. The material in which this increase took place consisted of 175000 grs. of water, 15635 grs. of copper, and 53370 grs. of lead, the whole having a capacity for heat equivalent to that of 178035 grs. of water. The volume of air passed through the pump was $12\cdot77\times26\times40=13280\cdot8$ cubic inches, which, at the observed barometric pressure and the temperature $54^{\circ}\cdot32$, would weigh 4252·7 grs. Hence we have for the specific heat—

$$\frac{178035\times0\cdot824}{4252\cdot7\times152\cdot136}=0\cdot22674.$$

By another series of experiments, in which the air-pump was worked at the velocity of twenty strokes per minute for twenty minutes, I obtained the value 0·2325. The mean of the three results is 0·22977, or nearly 0·23, which we may take as the specific heat of air at constant pressure determined by the above experiments.

Professor W. H. MILLER has remarked that MOLL's experiments, when correctly reduced, give a velocity of sound equal to 332·475 metres per second in dry air at 32°. Hence he deduces 1·41029 as the value of $k$. Calling it in round numbers 1·41, and the mechanical equivalent of heat 772, we obtain 0·238944 as the value of the specific heat of air at constant pressure, a result sufficiently near the experimental determination to show that the value of $k$, as deduced by Professor MILLER, is much nearer the truth than that upon which the tables of the foregoing paper are founded.

The values of $k$, as determined by the experiments of DESORMES and CLEMENT, GAY-LUSSAC and WELTER, and Mr. MEIKLE, referred to in the note to page 67, are respectively only 1·354, 1·375, and 1·333. In these experiments a small portion of air having been withdrawn from a large receiver, the equilibrium was re-established by opening for an instant a large aperture communicating with the external air, and then, after the receiver and its contents had regained their original temperature, the alteration of pressure, indicating the sudden rise of temperature which had taken place on the admission of the air, was noted. But it is obvious that the sudden admission of the air would cause the development of *sound*, and that, a portion of the *vis viva* escaping in this form, the increase of temperature and the deduced ratio of the specific heats would be diminished accordingly.

I subjoin Tables, similar to Tables I. and II., calculated from the data $k=1{\cdot}41$, and the specific heat of air at constant volume $=0{\cdot}169464$, or at constant pressure $=0{\cdot}238944$.

In Table IV., the examples 9, 10 and 11 may be suggested to the notice of the practical engineer, the temperature of the receiver being in all those cases below that of redness. I may remind the reader that the Table is founded on the supposition that the air which enters the pump has 491° of temperature from the absolute zero, and that its pressure is 15 lbs. on the square inch. If this initial temperature be altered, the whole of the other temperatures in the Table must be altered in the same proportion, but the pressure, work and economical duty will remain unchanged. If the initial pressure be altered, all the other pressures and work will suffer a proportionate change, but the temperatures and economical duty will remain the same. The above are obvious deductions from the formulæ on which the Tables are founded.

*Acton Square, Salford,*
*March* 20, 1852.

## Table III.

| Distance traversed by piston, in inches. | Work absorbed, in foot-pounds. | Temperature from absolute zero, in degrees Fahr. | Pressure on the piston, in pounds avoirdupois. | Distance traversed by piston, in inches. | Work absorbed, in foot-pounds. | Temperature from absolute zero, in degrees Fahr. | Pressure on the piston, in pounds avoirdupois. |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 491 | 15 | 6·0 | 12·025096 | 652·3847 | 39·86055 |
| 0·1 | 0·1257463 | 492·6876 | 15·17803 | 6·1 | 12·36122 | 656·8958 | 40·81647 |
| 0·2 | 0·2529680 | 494·3950 | 15·35970 | 6·2 | 12·70547 | 661·5159 | 41·81223 |
| 0·3 | 0·3817395 | 496·1232 | 15·54513 | 6·3 | 13·05820 | 666·2498 | 42·85023 |
| 0·4 | 0·5120683 | 497·8723 | 15·73438 | 6·4 | 13·41977 | 671·1023 | 43·93309 |
| 0·5 | 0·6439993 | 499·6429 | 15·92769 | 6·5 | 13·79055 | 676·0785 | 45·06354 |
| 0·6 | 0·7775396 | 501·4351 | 16·12503 | 6·6 | 14·17096 | 681·1838 | 46·24464 |
| 0·7 | 0·9127567 | 503·2498 | 16·32661 | 6·7 | 14·56146 | 686·4245 | 47·47966 |
| 0·8 | 1·049665 | 505·0872 | 16·53253 | 6·8 | 14·96245 | 691·8060 | 48·77217 |
| 0·9 | 1·188301 | 506·9478 | 16·74292 | 6·9 | 15·37448 | 697·3358 | 50·12596 |
| 1·0 | 1·328719 | 508·8323 | 16·95793 | 7·0 | 15·79807 | 703·0207 | 51·54529 |
| 1·1 | 1·471827 | 510·7529 | 17·17773 | 7·1 | 16·23377 | 708·8680 | 53·03472 |
| 1·2 | 1·615031 | 512·6748 | 17·40240 | 7·2 | 16·68220 | 714·8862 | 54·59923 |
| 1·3 | 1·761007 | 514·6339 | 17·63216 | 7·3 | 17·14397 | 721·0835 | 56·24433 |
| 1·4 | 1·908922 | 516·6190 | 17·86716 | 7·4 | 17·61984 | 727·4700 | 57·97600 |
| 1·5 | 2·058809 | 518·6306 | 18·10755 | 7·5 | 18·11050 | 734·0550 | 59·80080 |
| 1·6 | 2·210724 | 520·6694 | 18·35353 | 7·6 | 18·61680 | 740·8498 | 61·72605 |
| 1·7 | 2·364719 | 522·7361 | 18·60529 | 7·7 | 19·13958 | 747·8659 | 63·75969 |
| 1·8 | 2·520828 | 524·8312 | 18·86299 | 7·8 | 19·67980 | 755·1160 | 65·91058 |
| 1·9 | 2·679114 | 526·9555 | 19·12686 | 7·9 | 20·23844 | 762·6133 | 68·18854 |
| 2·0 | 2·839636 | 529·1098 | 19·39710 | 8·0 | 20·81664 | 770·3732 | 70·60445 |
| 2·1 | 3·002421 | 531·2945 | 19·67392 | 8·1 | 21·41559 | 778·4115 | 73·17045 |
| 2·2 | 3·167547 | 533·5106 | 19·95757 | 8·2 | 22·03660 | 786·7459 | 75·90002 |
| 2·3 | 3·335073 | 535·7589 | 20·24830 | 8·3 | 22·68110 | 795·3954 | 78·80838 |
| 2·4 | 3·505041 | 538·0400 | 20·54632 | 8·4 | 23·35061 | 804·3808 | 81·91249 |
| 2·5 | 3·677529 | 540·3549 | 20·85193 | 8·5 | 24·04690 | 813·7254 | 85·23164 |
| 2·6 | 3·852596 | 542·7044 | 21·16540 | 8·6 | 24·77180 | 823·4542 | 88·78743 |
| 2·7 | 4·030240 | 545·0885 | 21·48700 | 8·7 | 25·52742 | 833·5950 | 92·60451 |
| 2·8 | 4·210744 | 547·5110 | 21·81705 | 8·8 | 26·31602 | 844·1786 | 96·71086 |
| 2·9 | 4·393947 | 549·9697 | 22·15585 | 8·9 | 27·14016 | 855·2390 | 101·1385 |
| 3·0 | 4·580211 | 552·4695 | 22·50375 | 9·0 | 28·00265 | 866·8144 | 105·9244 |
| 3·1 | 4·769047 | 555·0038 | 22·86105 | 9·1 | 28·90667 | 878·9468 | 111·1106 |
| 3·2 | 4·961064 | 557·5808 | 23·22823 | 9·2 | 29·85575 | 891·6840 | 116·7460 |
| 3·3 | 5·156197 | 560·1996 | 23·60557 | 9·3 | 30·85385 | 905·0792 | 122·8892 |
| 3·4 | 5·354524 | 562·8613 | 23·99352 | 9·4 | 31·90550 | 919·1930 | 129·6058 |
| 3·5 | 5·556132 | 565·5670 | 24·39246 | 9·5 | 33·01577 | 934·0936 | 136·9750 |
| 3·6 | 5·761092 | 568·3177 | 24·80292 | 9·6 | 34·19049 | 949·8591 | 145·0905 |
| 3·7 | 5·969523 | 571·1150 | 25·22533 | 9·7 | 35·43632 | 966·5791 | 154·0638 |
| 3·8 | 6·181561 | 573·9607 | 25·66015 | 9·8 | 36·76094 | 984·3564 | 164·0289 |
| 3·9 | 6·397243 | 576·8553 | 26·10795 | 9·9 | 38·17335 | 1003·312 | 175·1490 |
| 4·0 | 6·616735 | 579·8010 | 26·56929 | 10·0 | 39·68387 | 1023·584 | 187·6224 |
| 4·1 | 6·840106 | 582·7988 | 27·04473 | 10·1 | 41·30480 | 1045·338 | 201·6945 |
| 4·2 | 7·067466 | 585·8501 | 27·53490 | 10·2 | 43·05077 | 1068·770 | 217·6720 |
| 4·3 | 7·299071 | 588·9584 | 28·04045 | 10·3 | 44·93904 | 1094·112 | 235·9413 |
| 4·4 | 7·534902 | 592·1234 | 28·56207 | 10·4 | 46·99081 | 1121·648 | 256·9966 |
| 4·5 | 7·775119 | 595·3475 | 29·10049 | 10·5 | 49·23177 | 1151·723 | 281·4802 |
| 4·6 | 8·019950 | 598·6331 | 29·65651 | 10·6 | 51·69402 | 1184·768 | 310·2390 |
| 4·7 | 8·269468 | 601·9818 | 30·23092 | 10·7 | 54·41741 | 1221·318 | 344·4105 |
| 4·8 | 8·523854 | 605·3958 | 30·82462 | 10·8 | 57·45355 | 1262·065 | 385·5594 |
| 4·9 | 8·783274 | 608·8774 | 31·43856 | 10·9 | 60·86887 | 1307·901 | 435·8856 |
| 5·0 | 9·047890 | 612·4287 | 32·07366 | 11·0 | 64·75245 | 1360·021 | 498·5821 |
| 5·1 | 9·317890 | 616·0523 | 32·73102 | 11·1 | 69·22596 | 1420·059 | 578·4345 |
| 5·2 | 9·593477 | 619·7509 | 33·41175 | 11·2 | 74·46110 | 1490·318 | 682·9350 |
| 5·3 | 9·874827 | 623·5267 | 34·11703 | 11·3 | 80·71017 | 1574·184 | 824·4195 |
| 5·4 | 10·16216 | 627·3830 | 34·84815 | 11·4 | 88·36276 | 1676·887 | 1024·575 |
| 5·5 | 10·45581 | 631·3240 | 35·60646 | 11·5 | 98·06077 | 1807·041 | 1324·918 |
| 5·6 | 10·75570 | 635·3486 | 36·39342 | 11·6 | 110·9605 | 1980·164 | 1814·815 |
| 5·7 | 11·06235 | 639·4641 | 37·21060 | 11·7 | 129·4314 | 2228·056 | 2722·671 |
| 5·8 | 11·37594 | 643·6727 | 38·05962 | 11·8 | 159·4567 | 2631·016 | 4822·635 |
| 5·9 | 11·69678 | 647·9786 | 38·94231 | 11·9 | 223·8930 | 3495·794 | 12815·505 |

## Table IV.

| No. of Example. | Receiver C. | | | Pump of Compresssion A. Length 12 inches. Sectional Area =1 square inch. | | | | Cylinder of Expansion B. Sectional Area =1 square inch. | | | | | Work evolved by the engine by each stroke of the piston, in foot-pounds. | Heat communicated to the air in receiver C, in degrees Fahr. per capacity of a lb. of water. | Work evolved out of each degree Fahr. in the capacity of a lb. of water, in foot-pounds. | Difference between the numbers in columns 4 and 13, divided by the numbers in column 4, and multiplied by the mechanical equivalent of heat. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Density of the air, that of the atmosphere being called unity. | Pressure of the air in pounds on the square inch. | Absolute temperature of the air, in degrees Fahr. from the absolute zero. | Length of the first part of the stroke. | Work of the engine absorbed by the first part of the stroke of the piston, in foot-pounds. | Work of the engine absorbed by the second part of the stroke of the piston, in foot-pounds. | Absolute temperature of the air forced into the receiver C, in degrees Fahr. from the absolute zero. | Length of the cylinder B, in inches. | Length of the first part of the piston's stroke, in inches. | Work communicated to the engine by the first part of the stroke, in foot-pounds. | Work communicated to the engine by the second part of the stroke, in foot-pounds. | Absolute temperature of the air escaping into the atmosphere, in degrees Fahr. from the absolute zero. | | | | |
| 1 | 1 | 26·56929 | 869·7014 | 4 | 1·616735 | 7·71286 | 579·801 | 18 | 12 | 11·56929 | 2·425102 | 736·5 | 4·664797 | 0·03945268 | 118·2378 | 118·2377 |
| 2 | 1 | 70·60445 | 2311·119 | 8 | 10·81664 | 18·53482 | 770·3732 | 36 | 12 | 55·60446 | 32·44992 | 1473·0 | 58·70292 | 0·2096809 | 279·9632 | 279·9630 |
| 3 | 2 | 70·60445 | 1155·559 | 8 | 10·81664 | 18·53482 | 770·3732 | 18 | 6 | 27·80223 | 16·22496 | 736·5 | 14·67573 | 0·05242022 | 279·9632 | 279·9628 |
| 4 | 2 | 187·6224 | 3070·753 | 10 | 27·18387 | 28·7704 | 1023·584 | 36 | 6 | 86·3112 | 81·55161 | 1473·0 | 111·90854 | 0·2786002 | 401·6815 | 401·6817 |
| 5 | 4 | 187·6224 | 1535·3765 | 10 | 27·18387 | 28·7704 | 1023·584 | 18 | 3 | 43·1556 | 40·7758 | 736·5 | 27·97713 | 0·06965 | 401·6815 | 401·6815 |
| 6 | 8 | 498·5821 | 2040·032 | 11 | 51·00245 | 40·29851 | 1360·021 | 18 | 1·5 | 60·44776 | 76·50367 | 736·5 | 45·65047 | 0·092543 | 493·2893 | 493·2897 |
| 7 | 20 | 1324·918 | 2168·449 | 11·5 | 83·68577 | 54·57992 | 1807·041 | 14·4 | 0·6 | 65·4959 | 100·4229 | 589·2 | 27·65313 | 0·04918419 | 562·2364 | 562·2361 |
| 8 | 100 | 12815·505 | 4194·953 | 11·9 | 209·018 | 106·6709 | 3495·794 | 14·4 | 0·12 | 128·0051 | 250·8216 | 589·2 | 63·1378 | 0·0951489 | 663·5685 | 663·5692 |
| 9 | 3 | 105·92437 | 1155·7525 | 9 | 16·75265 | 22·73109 | 866·8144 | 16 | 4 | 30·30812 | 22·33687 | 654·666 | 13·16125 | 0·03932171 | 334·7069 | 334·7069 |
| 10 | 2·5 | 105·92437 | 1386·903 | 9 | 16·75265 | 22·73109 | 866·8144 | 19·2 | 4·8 | 36·36974 | 26·80424 | 785·6 | 23·69024 | 0·0707791 | 334·7068 | 334·7068 |
| 11 | 4·5 | 187·6224 | 1364·779 | 10 | 27·18387 | 28·7704 | 1023·584 | 16 | $2\cdot\frac{2}{3}$ | 38·36053 | 36·24516 | 654·666 | 18·65142 | 0·0464334 | 401·6814 | 401·6816 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |

## *Additional Note on the preceding Paper. By* WILLIAM THOMSON, *M.A., F.R.S., F.R.S.E., Fellow of St. Peter's College, Cambridge, and Professor of Natural Philosophy in the University of Glasgow.*

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1. *Synthetical Investigation of the Duty of a Perfect Thermo-Dynamic Engine founded on the Expansions and Condensations of a Fluid, for which the gaseous laws hold and the ratio* (k) *of the specific heat under constant pressure to the specific heat in constant volume is constant*; *and modification of the result by the assumption of* MAYER'S *hypothesis.*

Let the source from which the heat is supplied be at the temperature S, and let T denote the temperature of the coldest body that can be obtained as a refrigerator. A cycle of the following four operations, *being reversible in every respect*, gives, according to CARNOT'S principle, first demonstrated for the Dynamical Theory by CLAUSIUS, the greatest possible statical mechanical effect that can be obtained in these circumstances from a quantity of heat supplied from the source.

(1.) Let a quantity of air contained in a cylinder and piston, at the temperature S, be allowed to expand to any extent, and let heat be supplied to it to keep its temperature constantly S.

(2.) Let the air expand farther, without being allowed to take heat from or to part with heat to surrounding matter, until its temperature sinks to T.

(3.) Let the air be allowed to part with heat so as to keep its temperature constantly T, while it is compressed to such an extent that at the end of the fourth operation the temperature may be S.

(4.) Let the air be farther compressed, and prevented from either gaining or parting with heat, till the piston reaches its primitive position.

The amount of mechanical effect gained on the whole of this cycle of operations will be the excess of the mechanical effect obtained by the first and second above the work spent in the third and fourth. Now if P and V denote the primitive pressure and volume of the air, and if $P_1$ and $V_1$, $P_2$ and $V_2$, $P_3$ and $V_3$, $P_4$ and $V_4$ denote the pressure and volume respectively, at the ends of the four successive operations, we have by the gaseous laws, and by POISSON'S formula and a conclusion from it quoted above, the following expressions:—

Mechanical effect obtained by the first operation $= PV \log \frac{V_1}{V}$.

Mechanical effect obtained by the second operation $= P_2V_2 . \frac{1}{k-1} . \left\{ \left( \frac{V_2}{V_1} \right)^{k-1} - 1 \right\}$.

Work spent in the third operation . . . . . . . $= P_3V_3 \log \frac{V_2}{V_3}$.

Work spent in the fourth operation . . . . . . $=P_3V_3 . \frac{1}{k-1}\left\{\left(\frac{V_3}{V_4}\right)^{k-1}-1\right\}$.

Now, according to the gaseous laws, we have

$$P_1V_1=PV; \quad P_2V_2=P_1V_1\frac{1+ET}{1+ES};$$

$$P_3V_3=P_2V_2; \quad \text{and (since } V_4=V\text{), } P_4=P.$$

Also by POISSON's formula,

$$\left(\frac{V_2}{V_1}\right)^{k-1}=\left(\frac{V_3}{V}\right)^{k-1}=\frac{1+ES}{1+ET}.$$

By means of these we perceive that the work spent in the fourth operation is equal to the mechanical effect gained in the second; and we find, for the whole gain of mechanical effect (denoted by M), the expressions

$$M=(PV-P_3V_3)\log\frac{V_1}{V}=PV\log\frac{V_1}{V}.\frac{E(S-T)}{1+ES}.$$

All the preceding formulæ are founded on the assumption of the gaseous laws and the constancy of the ratio ($k$) of the specific heat under constant pressure to the specific heat in constant volume, for the air contained in the cylinder and piston, and involve no other hypothesis*. If now we add the assumption of MAYER's hypothesis, which for the actual circumstance is $PV\log\frac{V_1}{V}=JH$, where H denotes the heat abstracted by the air from the surrounding matter in the first operation, and J the mechanical equivalent of a thermal unit, we have

$$M=JH.\frac{E(S-T)}{1+ES}.$$

The investigation of this formula given in my paper on the Dynamical Theory of Heat, shows that it would be true for every perfect thermo-dynamic engine, if MAYER's hypothesis were true for a fluid subject to the gaseous laws of pressure and density, whether, for such a fluid (did it exist), $k$ were constant or not.

It was first obtained by using, in the formula

$$M=JH\varepsilon^{-\frac{1}{J}\int_T^S \mu dt},$$

* From the sole hypothesis that $k$ is constant for a single fluid fulfilling the gaseous laws, and having E for its coefficient of expansion, I find it follows, as a necessary consequence, that CARNOT's function would have the form $\frac{JE}{1+Et+C}$; where C denotes an unknown absolute constant, and $t$ the temperature measured by a thermometer founded on the equable expansions of that gas. From this it follows, that for such a gas subjected to the four operations described in the text, we must have $PV\log\frac{V_1}{V}=JH\frac{1+ES}{1+ES+C}$, and consequently, $M=JH\frac{E(S-T)}{1+ES+C}$, which is Mr. RANKINE's general formula.

which involves no hypothesis, the expression

$$\mu = \frac{J}{\frac{1}{E} + t}$$

for CARNOT's function, which Mr. JOULE had suggested to me in a letter dated December 9, 1848, as the expression of MAYER's hypothesis, in terms of the notation of my "Account of CARNOT's Theory.*" Mr. RANKINE† has arrived at a formula agreeing with it (with the exception of a constant term in the denominator, which, as its value is unknown, but probably small, he neglects in the actual use of the formula), as a consequence of the fundamental principles assumed in his Theory of Molecular Vortices, when applied to any fluid whatever, experiencing a cycle of four operations satisfying CARNOT's criterion of reversibility (being in fact precisely analogous to those described above, and originally invented by CARNOT); and he thus establishes CARNOT's law as a consequence of the equations of the mutual conversion of heat and expansive power, which had been given in the first section of his paper on the Mechanical Action of Heat‡.

## 2. *Note on the Specific Heats of Air.*

Let N be the specific heat of unity of weight of a fluid at the temperature $t$, kept within constant volume, $v$; and let $k$N be the specific heat of the same fluid mass, under constant pressure, $p$. Without any other assumption than that of CARNOT's principle, the following equation is demonstrated in my paper§ on the Dynamical Theory of Heat, § 48,

$$kN - N = \frac{\left(\frac{dp}{dt}\right)^2}{\mu \times -\frac{dp}{dv}},$$

where $\mu$ denotes the value of CARNOT's function, for the temperature $t$, and the differentiations indicated are with reference to $v$ and $t$ considered as independent variables, of which $p$ is a function. If the fluid be subject to BOYLE's and MARIOTTE's law of compression, we have

$$\frac{dp}{dv} = -\frac{p}{v};$$

and if it be subject also to GAY-LUSSAC's law of expansion,

$$\frac{dp}{dt} = \frac{Ep}{1+Et}.$$

* Royal Society of Edinburgh, January 2, 1849, Transactions, vol. xvi. part 5.

† On the Economy of Heat in Expansive Engines. Royal Society of Edinburgh, April 21, 1851, Transactions, vol. xx. part 2.

‡ Royal Society of Edinburgh, February 4, 1850, Transactions, vol. xx. part 1.

§ Royal Society of Edinburgh, March 17, 1851, Transactions, vol. xx. part 2.

Hence, for such a fluid,

$$k\mathrm{N}-\mathrm{N}=\frac{\mathrm{E}^2 pv}{\mu(1+\mathrm{E}t)^2}\ *.$$

In the case of dry air these laws are fulfilled to a very high degree of approximation, and, for it, according to REGNAULT's observations,

$$\frac{pv}{1+\mathrm{E}t}=26215, \quad \mathrm{E}=\cdot 00366$$

(a British foot being the unit of length, and the weight of a British pound at Paris, the unit of force).

We have consequently, for dry air,

$$k\mathrm{N}-\mathrm{N}=\frac{26215\mathrm{E}^2}{\mu(1+\mathrm{E}t)} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Now it is demonstrated, without any other assumption than that of CARNOT's principle, in my "Account of CARNOT's Theory" (Appendix III.), that

$$\frac{\mathrm{E}}{\mu(1+\mathrm{E}t)}=\frac{\mathrm{H}}{\mathrm{W}},$$

if W denote the quantity of work that must be spent in compressing a fluid subject to the gaseous laws, to produce H units of heat when its temperature is kept at $t$. Hence

$$k\mathrm{N}-\mathrm{N}=26215\mathrm{E}\times\frac{\mathrm{H}}{\mathrm{W}}=95\cdot 947\times\frac{\mathrm{H}}{\mathrm{W}} \quad \ldots\ldots\ldots \quad (2)$$

If we adopt the values of $\mu$ shown in Table I. of the "Account of CARNOT's Theory," depending on no uncertain data except the densities of saturated steam at different temperatures, which, for want of accurate experimental data, were derived from the value $\frac{1}{1693\cdot 5}$ for the density of saturated vapour at 100°, by the assumption of the "gaseous laws" of variation with temperature and pressure; we find 1357 and 1369 for the values of $\frac{\mathrm{E}}{\mu(1+\mathrm{E}t)}$ at the temperatures 0 and 10° respectively; and hence, for these temperatures,

$$\left.\begin{array}{ll}(t=0) & k\mathrm{N}-\mathrm{N}=\frac{95\cdot 947}{1357}=\cdot 07071 \\ (t=10^\circ) & k\mathrm{N}-\mathrm{N}=\frac{95\cdot 947}{1369}=\cdot 07008\end{array}\right\} \quad \ldots\ldots\ldots \quad (a).$$

Or, if we adopt MAYER's hypothesis, according to which $\frac{\mathrm{W}}{\mathrm{H}}$ is equal to the mechanical equivalent of the thermal unit†, we have $\frac{\mathrm{W}}{\mathrm{H}}=1390$; and hence, for all temperatures,

$$k\mathrm{N}-\mathrm{N}=\frac{95\cdot 947}{1390}=\cdot 06903 \quad \ldots\ldots\ldots\ldots \quad (a').$$

* This equation expresses a proposition first demonstrated by CARNOT. See "Account of CARNOT's Theory," Appendix III. (Transactions Royal Society of Edinburgh, vol. xvi. part 5.)

† The number 1390, derived from Mr. JOULE's experiments on the friction of fluids, cannot differ by $\frac{1}{100}$, and probably does not differ by $\frac{1}{300}$, of its own value, from the true value of the mechanical equivalent of the thermal unit.

The very accurate observations which have been made on the velocity of sound in air, taken in connection with the results of REGNAULT's observations on its density, &c., lead to the value 1·410 for $k$, which is probably true in three if not in four of its figures. Now, $k$ being known, the preceding equations enable us to determine the absolute values of the two specific heats ($k$N, and N) according to the hypotheses used in ($a$) and in ($a'$) respectively; and we thus find,

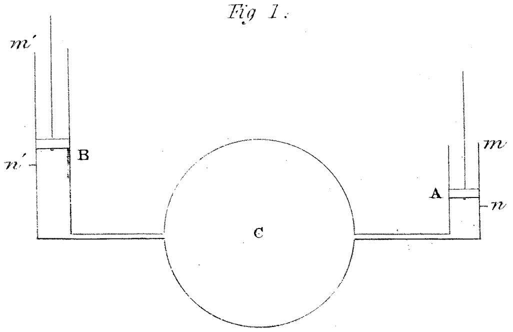
| | Specific heat of air under constant pressure ($k$N). | Specific heat of air in constant volume (N). | |
|---|---|---|---|
| for $t=0$, | ·2431 | ·1724, | according to the tabulated values of CARNOT's function. |
| for $t=10$, | ·2410 | ·1709, | |
| Or, for all temperatures, | ·2374 | ·1684, | according to MAYER's hypothesis. |

By the adoption of hypotheses involving that of MAYER, and taking 1389·6 and 1·4 as the values of J and $k$, respectively, Mr. RANKINE finds ·2404 and ·1717 as the values of the two specific heats.

Hence it is probable that the values of the specific heat of air under constant pressure, found by SUERMANN (·3046), and by DE LA ROCHE and BERARD (·2669), are both considerably too great; and the true value, to two significant figures, is probably ·24.

*Glasgow College,*
*February* 19, 1852.

Fig 1.



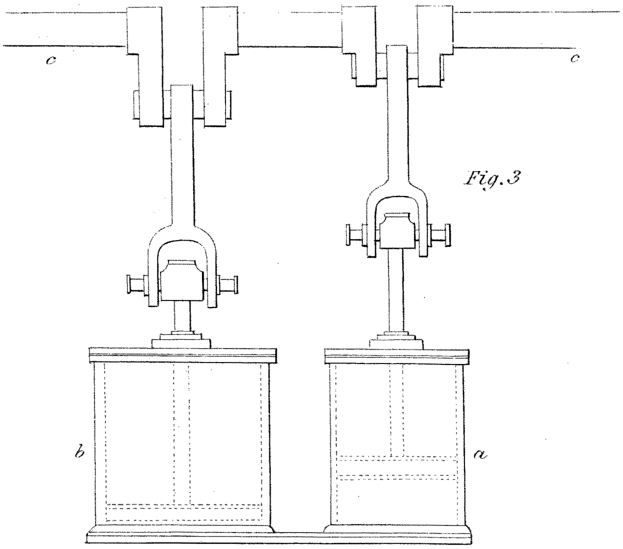
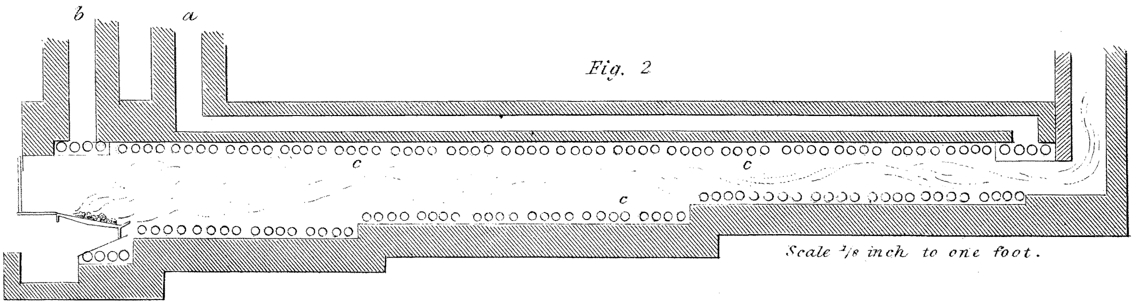



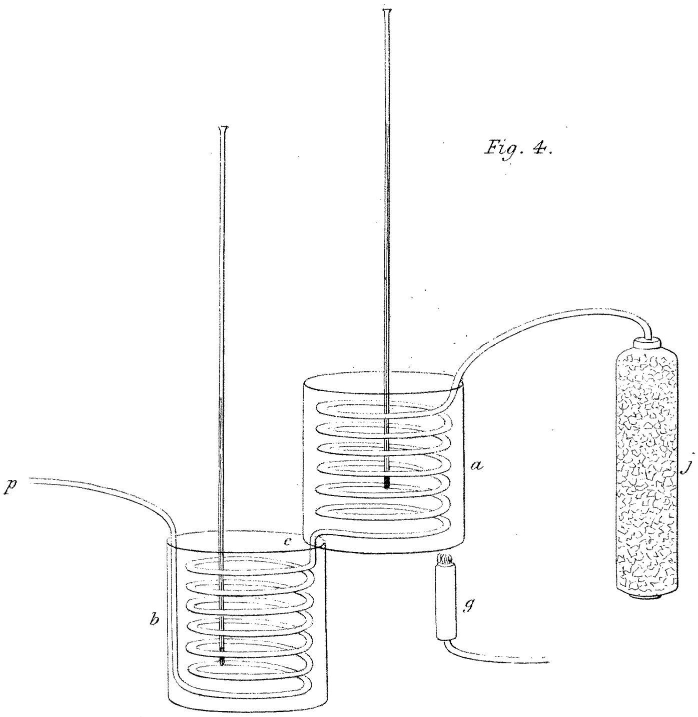

Fig. 3

Fig. 2



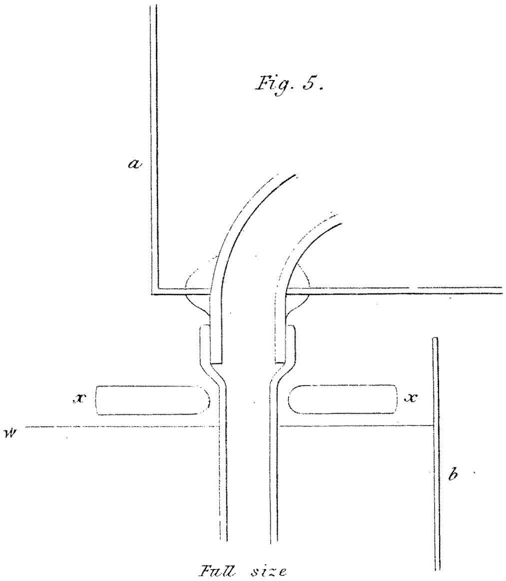

Scale 1/8 inch to one foot.

Fig. 4.




Scale 1/12

Fig. 5.

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